

定義

$$f_x(a, b) = \lim_{h \rightarrow 0} \frac{f(a+h, b) - f(a, b)}{h}$$

$$f_y(a, b) = \lim_{h \rightarrow 0} \frac{f(a, b+h) - f(a, b)}{h}$$

問題 2.4

$$f(x, y) = \begin{cases} \frac{x \cdot |y|}{\sqrt{x^2 + y^2}} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

定義より

$$f_x(0, 0) = \lim_{h \rightarrow 0} \frac{f(0+h, 0) - f(0, 0)}{h} = 0$$

$$\Rightarrow f(h, 0) = \frac{h \cdot |0|}{\sqrt{h^2 + 0^2}} = 0$$

$$\therefore f_x(0, 0) = \lim_{h \rightarrow 0} \frac{0}{h} = 0$$

同様にして

$$f_y(0, 0) = \lim_{h \rightarrow 0} \frac{f(0, 0+h) - f(0, 0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{0 - 0}{h} = 0$$