

線形代数 第6回小テスト 解答

1

(1) $\varphi_1^{-1} = \varphi_1, \varphi_2^{-1} = \varphi_3, \varphi_3^{-1} = \varphi_2, \varphi_4^{-1} = \varphi_4, \varphi_5^{-1} = \varphi_5, \varphi_6^{-1} = \varphi_6.$

(2) $\varphi_1\varphi_2 = \varphi_2, \varphi_2\varphi_2 = \varphi_3, \varphi_3\varphi_2 = \varphi_1, \varphi_4\varphi_2 = \varphi_5, \varphi_5\varphi_2 = \varphi_6, \varphi_6\varphi_2 = \varphi_4.$

2

$\psi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} = (1, 2)(2, 4)$ と 2 つの互換の積として表せるので, $\text{sgn}(\psi) = 1.$

3

(1) $\det \begin{pmatrix} 3 & -2 \\ 4 & -3 \end{pmatrix} = 3 \times (-3) - (-2) \times 4 = -9 + 8 = \underline{-1}$

(2) $\det \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = 1 \times 1 - (-1) \times (-1) = 1 - 1 = \underline{0}$

(3) $\det \begin{pmatrix} 2 & 1 & -1 \\ -1 & 1 & 3 \\ 1 & 0 & -2 \end{pmatrix}$
 $= 2 \times 1 \times (-2) + 1 \times 3 \times 1 + (-1) \times (-1) \times 0 - (-1) \times 1 \times 1 - 2 \times 3 \times 0 - 1 \times (-1) \times (-2)$
 $= -4 + 3 + 0 + 1 + 0 - 2$
 $= \underline{-2}$

(4) $\det \begin{pmatrix} 1 & 1 & -1 \\ -1 & 1 & 2 \\ 4 & 2 & 1 \end{pmatrix}$
 $= 1 \times 1 \times 1 + 1 \times 2 \times 4 + (-1) \times (-1) \times 2 - (-1) \times 1 \times 4 - 1 \times 2 \times 2 - 1 \times (-1) \times 1$
 $= 1 + 2 + 8 + 4 - 4 + 1$
 $= \underline{12}$