

[1] $f(x) = \frac{x^4 - x^3 + x^2 - x + 1}{x^3 - x^2 + x - 1}$

$$(1) f(x) = x + \frac{1}{x^3 - x^2 + x - 1}$$

$$(2) x^3 - x^2 + x - 1 = (x - 1)(x^2 + 1)$$

$$(3) \frac{1}{x^3 - x^2 + x - 1} = \frac{1}{2} \times \frac{1}{x - 1} - \frac{1}{2} \times \frac{x + 1}{x^2 + 1}$$

(4)

$$\begin{aligned} \int f(x) dx &= \int x dx + \frac{1}{2} \int \frac{1}{x - 1} dx - \frac{1}{2} \int \frac{x + 1}{x^2 + 1} dx \\ &= \frac{1}{2} x^2 + \frac{1}{2} \log|x - 1| - \frac{1}{4} \int \frac{(x^2 + 1)'}{x^2 + 1} dx - \frac{1}{2} \int \frac{1}{x^2 + 1} dx \\ &= \frac{1}{2} x^2 + \frac{1}{2} \log|x - 1| - \frac{1}{4} \log(x^2 + 1) - \frac{1}{2} \arctan x + C \end{aligned}$$

[2] ヒント：

$$I_n = \int (\log x)^n dx = \int (x)' (\log x)^n dx$$