問題 6.3.

(i) 方程式 $3x+2y+2z=-1$ が $\tilde{z}=0$ となるように座標変換する[*1]．たとえば,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = P\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} + \vec{v} = \begin{pmatrix} \frac{4}{\sqrt{34}} & 0 & \frac{3}{\sqrt{17}} \\ -\frac{3}{\sqrt{34}} & \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{17}} \\ -\frac{3}{\sqrt{34}} & -\frac{1}{\sqrt{2}} & \frac{2}{\sqrt{17}} \end{pmatrix}\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

(ii) 視点 S を同次座標で表し，さらに $\tilde{x}\tilde{y}\tilde{z}$-座標に変換する；

$$S = \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix} = \begin{bmatrix} 2 \\ 3 \\ 7 \\ 1 \end{bmatrix} \xrightarrow{\text{座標変換}} \tilde{S} = \left(\begin{array}{ccc|c} & P & & \vec{v} \\ \hline 0 & 0 & 0 & 1 \end{array}\right)^{-1}\begin{bmatrix} 2 \\ 3 \\ 7 \\ 1 \end{bmatrix}$$

$$= \left(\begin{array}{ccc|c} & {}^tP & & -{}^tP\vec{v} \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\begin{bmatrix} 2 \\ 3 \\ 7 \\ 1 \end{bmatrix}$$

$$= \left(\begin{array}{ccc|c} \frac{4}{\sqrt{34}} & -\frac{3}{\sqrt{34}} & -\frac{3}{\sqrt{34}} & \frac{7}{\sqrt{34}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{3}{\sqrt{17}} & \frac{2}{\sqrt{17}} & \frac{2}{\sqrt{17}} & \frac{1}{\sqrt{17}} \\ \hline 0 & 0 & 0 & 1 \end{array}\right)\begin{bmatrix} 2 \\ 3 \\ 7 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{15}{\sqrt{34}} \\ -\frac{5}{\sqrt{2}} \\ \frac{27}{\sqrt{17}} \\ 1 \end{bmatrix} = \begin{bmatrix} -15 \\ -5\sqrt{17} \\ 27\sqrt{2} \\ \sqrt{34} \end{bmatrix}$$

(iii) $\tilde{z}=0$ への透視投影（視点 $\tilde{S}$）を表す行列をつくる；

$$\varphi_{\tilde{S}} = \begin{pmatrix} -27\sqrt{2} & 0 & -15 & 0 \\ 0 & -27\sqrt{2} & -5\sqrt{17} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{34} & -27\sqrt{2} \end{pmatrix}$$

[*1] 12 月 7 日の講義メモおよび自身のノートを参照.

(iv) xyz-座標における π への透視投影を表す行列を計算する；

$$
\begin{aligned}
\Phi_S &= \left(\begin{array}{ccc|c} & P & & \vec{v} \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \times \varphi_{\tilde{S}} \times \left(\begin{array}{ccc|c} & {}^tP & & -{}^tP\vec{v} \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \\
&= \left(\begin{array}{ccc|c} \frac{4}{\sqrt{34}} & 0 & \frac{3}{\sqrt{17}} & -1 \\ -\frac{3}{\sqrt{34}} & \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{17}} & 1 \\ -\frac{3}{\sqrt{34}} & -\frac{1}{\sqrt{2}} & \frac{2}{\sqrt{17}} & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \begin{pmatrix} -27\sqrt{2} & 0 & -15 & 0 \\ 0 & -27\sqrt{2} & -5\sqrt{17} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{34} & -27\sqrt{2} \end{pmatrix} \\
&\qquad \times \left(\begin{array}{ccc|c} \frac{4}{\sqrt{34}} & -\frac{3}{\sqrt{34}} & -\frac{3}{\sqrt{34}} & \frac{7}{\sqrt{34}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{3}{\sqrt{17}} & \frac{2}{\sqrt{17}} & \frac{2}{\sqrt{17}} & \frac{1}{\sqrt{17}} \\ \hline 0 & 0 & 0 & 1 \end{array}\right) \\
&= \sqrt{2} \begin{pmatrix} -21 & 4 & 4 & 2 \\ 9 & -21 & 6 & 3 \\ 21 & 14 & -13 & 7 \\ 3 & 2 & 2 & -26 \end{pmatrix}
\end{aligned}
$$

(v) 各点 A, B, C, D, E, F を同次座標で表し，行列 Φ_S をかける；

$$
\Phi_S(A) = \begin{bmatrix} -3\sqrt{2} \\ 9\sqrt{2} \\ 3\sqrt{2} \\ -15\sqrt{2} \end{bmatrix}, \quad \Phi_S(B) = \begin{bmatrix} 39\sqrt{2} \\ -9\sqrt{2} \\ -39\sqrt{2} \\ -21\sqrt{2} \end{bmatrix}, \quad \Phi_S(C) = \begin{bmatrix} 31\sqrt{2} \\ 33\sqrt{2} \\ -67\sqrt{2} \\ -25\sqrt{2} \end{bmatrix},
$$

$$
\Phi_S(D) = \begin{bmatrix} -11\sqrt{2} \\ 51\sqrt{2} \\ -25\sqrt{2} \\ -19\sqrt{2} \end{bmatrix}, \quad \Phi_S(E) = \begin{bmatrix} 8\sqrt{2} \\ 12\sqrt{2} \\ -\frac{25\sqrt{2}}{2} \\ -23\sqrt{2} \end{bmatrix}, \quad \Phi_S(F) = \begin{bmatrix} 20\sqrt{2} \\ 30\sqrt{2} \\ -\frac{103\sqrt{2}}{2} \\ -17\sqrt{2} \end{bmatrix}.
$$

(vi) 各点の Φ_S による像を直交座標で表す；

$$
\Phi_S(A) = \begin{pmatrix} \frac{1}{5} \\ -\frac{3}{5} \\ -\frac{1}{5} \end{pmatrix}, \quad \Phi_S(B) = \begin{pmatrix} -\frac{13}{7} \\ \frac{3}{7} \\ \frac{13}{7} \end{pmatrix}, \quad \Phi_S(C) = \begin{pmatrix} -\frac{31}{25} \\ -\frac{33}{25} \\ \frac{67}{25} \end{pmatrix},
$$

$$
\Phi_S(D) = \begin{pmatrix} \frac{11}{19} \\ -\frac{51}{19} \\ \frac{25}{19} \end{pmatrix}, \quad \Phi_S(E) = \begin{pmatrix} -\frac{8}{23} \\ -\frac{12}{23} \\ \frac{25}{46} \end{pmatrix}, \quad \Phi_S(F) = \begin{pmatrix} -\frac{20}{17} \\ -\frac{30}{17} \\ \frac{103}{34} \end{pmatrix}.
$$