

問題 1.1. (1) $504 = 2^3 \times 3^2 \times 7$ (2) $143 = 11 \times 13$ (3) $256 = 2^8$

問題 1.2. (1) $24 = 2^3 \times 3$, $28 = 2^2 \times 7$. 最小公倍数は $2^3 \times 3 \times 7 = 168$, 最大公約数は $2^2 = 4$.

(2) $10 = 2 \times 5$, $21 = 3 \times 7$. 最小公倍数は $2 \times 3 \times 5 \times 7 = 210$, 最大公約数は 1.

(3) $70 = 2 \times 5 \times 7$, $360 = 2^3 \times 3^2 \times 5$. 最小公倍数は $2^3 \times 3^2 \times 5 \times 7 = 2520$, 最大公約数は $2 \times 5 = 10$.

定理

自然数 a と b の最大公約数を $\text{GCD}(a, b)$, 最小公倍数を $\text{LCM}(a, b)$ と書く. このとき, 次の等式が成り立つ.

$$ab = \text{GCD}(a, b) \times \text{LCM}(a, b)$$

問題 1.3. (ア) (イ) (ウ) の 3 つ. 循環小数は有理数である (整数の比で書ける).

問題 1.4. (1) $|-5| = 5$

(2) $|13| = 13$

(3) $|\pi - 3| = \pi - 3$

(4) $|\sqrt{2} - 2| = 2 - \sqrt{2}$

(5) $|\sqrt{2} - 1| + |\sqrt{2} - 4| = (\sqrt{2} - 1) - (\sqrt{2} - 4) = 3$

問題 1.5. (1) $2\sqrt{5} \times 3\sqrt{20} = (2 \times 3)\sqrt{5 \times 20} = 6\sqrt{5^2 \times 2^2} = 6 \times 5 \times 2 = 60$

(2) $3\sqrt{27} + 2\sqrt{12} - \sqrt{75} = 9\sqrt{3} + 4\sqrt{3} - 5\sqrt{3} = (9 + 4 - 5)\sqrt{3} = 8\sqrt{3}$

(3) $\sqrt{5}(\sqrt{40} - 4\sqrt{5}) = \sqrt{5}(2\sqrt{10} - 4\sqrt{5}) = 2\sqrt{50} - 4 \times 5 = 10\sqrt{2} - 20$

(4) $(2\sqrt{3} - 5)(\sqrt{3} + 3) = 2 \times 3 + 6\sqrt{3} - 5\sqrt{3} - 15 = \sqrt{3} - 9$

(5) $(\sqrt{5} - 3)(\sqrt{5} + 3) = (\sqrt{5})^2 - 3^2 = 5 - 9 = -4$

(6) $(\sqrt{5} + \sqrt{2})^2 = 5 + 2(\sqrt{5} \times \sqrt{2}) + 2 = 7 + 2\sqrt{10}$

(7) $\frac{\sqrt{50}}{\sqrt{8}} = \sqrt{\frac{50}{8}} = \sqrt{\frac{25}{4}} = \sqrt{\frac{5^2}{2^2}} = \frac{5}{2}$

問題 1.6. (1) $\frac{14}{3\sqrt{7}} = \frac{2\sqrt{7}}{3}$ (2) $\frac{1}{\sqrt{3} + 1} = \frac{1 \times (\sqrt{3} - 1)}{(\sqrt{3} + 1) \times (\sqrt{3} - 1)} = \frac{\sqrt{3} - 1}{2}$

(3) $\frac{1 + \sqrt{2}}{1 - \sqrt{2}} = \frac{(1 + \sqrt{2}) \times (1 + \sqrt{2})}{(1 - \sqrt{2}) \times (1 + \sqrt{2})} = -3 - 2\sqrt{2}$