科 目 名　応用数学 I-J　　出 題 者 名　佐藤 弘康

1　次の累次積分を求めなさい.

【各 4 点】

(1) $\displaystyle\int_1^2 \int_1^3 (2x-y)\,dx\,dy$

$$\begin{aligned}
&= \int_1^2 \int_1^3 (2x-y)\,dx\,dy \\
&= \int_1^2 \left[x^2 - xy\right]_{x=1}^{x=3} dy \\
&= \int_1^2 \{(9-3y)-(1-y)\}\,dy \\
&= \int_1^2 (8-2y)\,dy \\
&= \left[8y - y^2\right]_1^2 \\
&= 16 - 4 - (8-1) \\
&= \mathbf{5}.
\end{aligned}$$

(2) $\displaystyle\int_0^1 \int_{-x}^{2x} x^2 y^2\,dy\,dx$

$$\begin{aligned}
&= \int_0^1 x^2 \left[\frac{y^3}{3}\right]_{y=-x}^{y=2x} dx \\
&= \int_0^1 \frac{x^2}{3}\{8x^3 - (-x)^3\}dx \\
&= \int_0^1 \frac{x^2}{3}\cdot 9x^3\,dx \\
&= \int_0^1 3x^5\,dx \\
&= 3\left[\frac{x^6}{6}\right]_0^1 \\
&= \mathbf{\frac{1}{2}}.
\end{aligned}$$

(3) $\displaystyle\int_0^2 \int_0^{2x} e^{x-y}\,dy\,dx$

$$\begin{aligned}
&= \int_0^2 e^x \int_0^{2x} e^{-y}\,dy\,dx \\
&= \int_0^2 e^x \left[-e^{-y}\right]_{y=0}^{y=2x} dx \\
&= \int_0^2 e^x \left(-e^{-2x}+1\right) dx \\
&= \int_0^2 \left(e^x - e^{-x}\right) dx \\
&= \left[e^x + e^{-x}\right]_0^2 \\
&= e^2 + e^{-2} - (1+1) \\
&= \mathbf{e^2 + e^{-2} - 2 = (e - e^{-1})^2}.
\end{aligned}$$

2　次の 2 重積分を累次積分の形に直しなさい.

【各 4 点】

(1) $\displaystyle\iint_D f(x,y)\,dxdy \quad D: 0 \leqq x \leqq 1,\ 1 \leqq y \leqq 2$

$$= \int_1^2 \int_0^1 f(x,y)\,dx\,dy = \int_0^1 \int_1^2 f(x,y)\,dy\,dx.$$

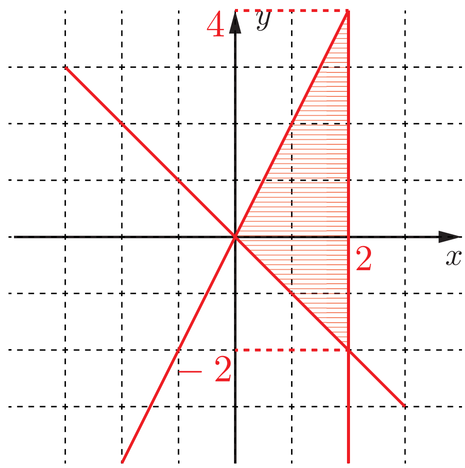
(2) $\displaystyle\iint_D f(x,y)\,dxdy \quad D: y \leqq x \leqq 2y,\ 0 \leqq y \leqq 2$

$$= \int_0^2 \int_y^{2y} f(x,y)\,dx\,dy.$$

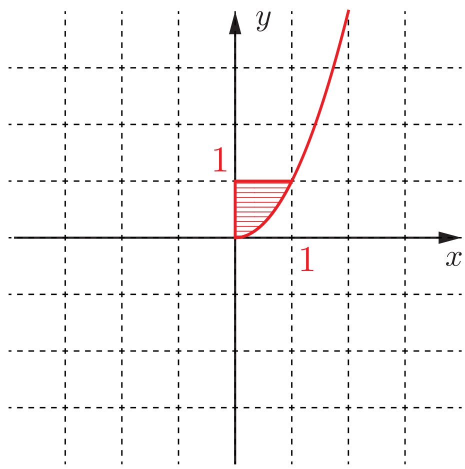
3　次の 2 つの不等式が表す領域 D を xy-平面に図示しなさい.

【各 4 点】

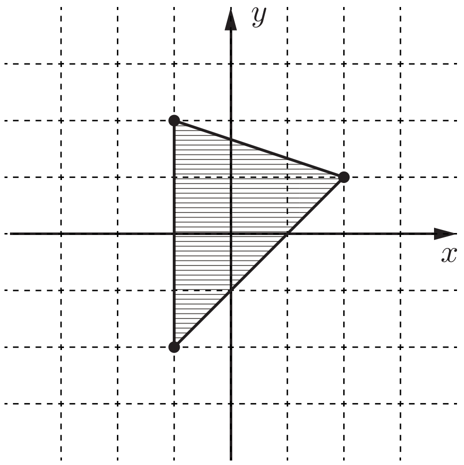
(1) $D: 0 \leqq x \leqq 2,\ -x \leqq y \leqq 2x$



(2) $D: 0 \leqq x \leqq \sqrt{y},\ 0 \leqq y \leqq 1$



4 3 点 $(-1, 2)$, $(-1, -2)$, $(2, 1)$ を頂点とする三角形の領域（下図参照）を x, y の不等式で表しなさい．



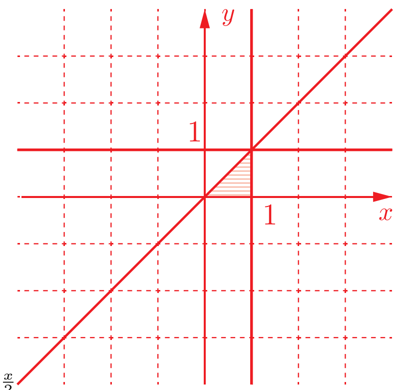
$-1 \leqq x \leqq 2, x - 1 \leqq y \leqq \dfrac{5}{3} - \dfrac{x}{3}$　　【4 点】

5 次の累次積分の積分順序を変更しなさい．

【各 4 点】

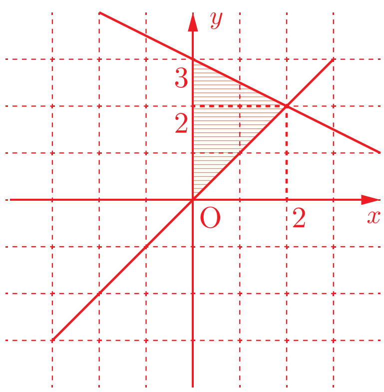
(1) $\displaystyle\int_0^1 \int_0^x f(x, y)\, dy\, dx$

$$= \int_0^1 \int_y^1 f(x, y)\, dx\, dy.$$



(2) $\displaystyle\int_0^2 \int_x^{3-\frac{x}{2}} f(x, y)\, dy\, dx$

$$= \int_0^2 \int_0^y f(x, y)\, dx\, dy + \int_2^3 \int_0^{6-2y} f(x, y)\, dx\, dy.$$



6 次の不等式で表される空間の領域の体積 V を求めなさい．

【各 5 点（体積を累次積分として書けていれば 1 点)】

(1) $V : 1 \leqq x \leqq 2, \quad 1 \leqq y \leqq 3, \quad 0 \leqq z \leqq 2x + y^2$

領域 $D : 1 \leqq x \leqq 2, 1 \leqq y \leqq 3$ 上で $2x + y^2 \geqq 0$ であるから,

$$\begin{aligned}
V &= \int_1^2 \int_1^3 (2x + y^2)\, dy\, dx \\
&= \int_1^2 \left[2xy + \frac{y^3}{3}\right]_{y=1}^{y=3} dx \\
&= \int_1^2 \left\{(6x + 9) - \left(2x + \frac{1}{3}\right)\right\} dx \\
&= \int_1^2 \left(4x + \frac{26}{3}\right) dx \\
&= \left[2x^2 + \frac{26}{3}x\right]_1^2 dx \\
&= 8 + \frac{52}{3} - \left(2 + \frac{26}{3}\right) \\
&= \mathbf{\frac{44}{3}}.
\end{aligned}$$

(2) $V : 0 \leqq x \leqq 1, \quad -x \leqq y \leqq 0, \quad 0 \leqq z \leqq (x + y)e^y$

$-x \leq y$ を満たすので, $(x+y)e^y \geqq (x+(-x))e^x = 0$. よって,

$$\begin{aligned}
V &= \int_0^1 \int_{-x}^0 (x + y)e^y\, dy\, dx \\
&= \int_0^1 \int_{-x}^0 (x + y)\,(e^y)'\, dy\, dx \\
&= \int_0^1 \left\{[(x + y)e^y]_{y=-x}^{y=0} - \int_{-x}^0 (x + y)' e^y\, dy\right\} dx \\
&= \int_0^1 \left\{x - \int_{-x}^0 e^y\, dy\right\} dx \\
&= \int_0^1 \left\{x - [e^y]_{y=-x}^{y=0}\right\} dx \\
&= \int_0^1 (x - 1 + e^{-x})\, dx \\
&= \left[\frac{x^2}{2} - x - e^{-x}\right]_0^1 \\
&= \frac{1}{2} - 1 - e^{-1} - (-1) \\
&= \mathbf{\frac{1}{2} - \frac{1}{e}}.
\end{aligned}$$