科 目 名　微分積分学Ⅰ　　出 題 者 名　佐藤 弘康

**1** 次の極限値を求めなさい．

(1) $\lim_{x\to -1}\dfrac{x+1}{x^2+4x+3}$ $= \lim_{x\to -1}\dfrac{x+1}{(x+3)(x+1)}$

$= \lim_{x\to -1}\dfrac{\cancel{x+1}}{(x+3)\cancel{(x+1)}}$

$= \lim_{x\to -1}\dfrac{1}{x+3} = \dfrac{1}{-1+3}$

$= \dfrac{1}{2}$

(2) $\lim_{x\to 2}\dfrac{x^2-6x+8}{x^2+x-6}$

$= \lim_{x\to 2}\dfrac{(x-2)(x-4)}{(x-2)(x+3)}$

$= \lim_{x\to 2}\dfrac{x-4}{x+3} = \dfrac{2-4}{2+3} = -\dfrac{2}{5}$

(3) $\lim_{x\to 1}\dfrac{\sqrt{x+1}-\sqrt{2}}{x-1}$

$= \lim_{x\to 1}\dfrac{(\sqrt{x+1}-\sqrt{2})(\sqrt{x+1}+\sqrt{2})}{(x-1)(\sqrt{x+1}+\sqrt{2})}$

$= \lim_{x\to 1}\dfrac{(x+1)-2}{(x-1)(\sqrt{x+1}+\sqrt{2})}$

$= \lim_{x\to 1}\dfrac{1}{\sqrt{x+1}+\sqrt{2}} = \dfrac{1}{\sqrt{1+1}+\sqrt{2}} = \dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{2}}{4}$

**2** 導関数の定義にしたがって，関数 $y=\sqrt{x}$ を微分しなさい．

$f(x) = \sqrt{2}$ とおく

$f'(x) = \lim_{\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

$= \lim_{h\to 0}\dfrac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0}\dfrac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0}\dfrac{1}{\sqrt{x+h}+\sqrt{x}} = \dfrac{1}{\sqrt{x+0}+\sqrt{x}} = \dfrac{1}{2\sqrt{x}}$

**3** 次の関数を微分しなさい．

(1) $y = 3x^4-2x^3+5x+3$

$y' = 12x^3 - 6x^2 + 5$

(2) $y = (3-2x)^3$

$$y' = 3(3-2x)^{3-1} \times (-2)$$
$$= -6(3-2x)^2$$

(3) $y = \dfrac{1}{x+1}$

$$y' = \frac{-1}{(x+1)^2} = -\frac{1}{(x+1)^2}$$

(4) $y = \dfrac{3-x}{x+7}$

$$y' = \frac{(x+7)\times(-1) - (3-x)\times 1}{(x+7)^2}$$
$$= \frac{-x-7+x-3}{(x+7)^2} = -\frac{10}{(x+7)^2}$$

(5) $y = \dfrac{2}{x} - \dfrac{1}{x^2}$ $= 2x^{-1} - x^{-2}$

$$y' = 2\times(-1)x^{-1-1} - (-2)x^{-2-1}$$
$$= -2x^{-2} + 2x^{-3} = \frac{2}{x^3}(1-x)$$

(6) $y = (x^2+2)\sqrt{2x-1}$

$$y' = (x^2+2)'\sqrt{2x-1} + (x^2+2)\left(\sqrt{2x-1}\right)'$$
$$= 2x\sqrt{2x-1} + (x^2+2)\times\frac{1}{2}(2x-1)^{\frac{1}{2}-1}\times 2$$
$$= 2x\sqrt{2x-1} + \frac{x^2+2}{\sqrt{2x-1}}$$
$$= \frac{2x(2x-1)+x^2+2}{\sqrt{2x-1}}$$
$$= \frac{5x^2-2x+2}{\sqrt{2x-1}}$$

学　科

学籍番号

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